Handling uncertainty in higher dimensions with potential clouds towards robust design optimization

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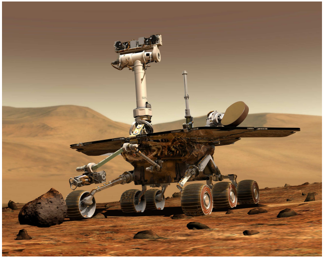
Overview

- Motivation
- Uncertainty modeling with potential clouds
- Design optimization problem formulation
- Application: Mars Exploration Rover



Problems

- Curse of dimensionality
  - severe computational effort
  - high dimensionality of many real life problems
- Incomplete information
  - scarce data, conflicting, or unformalized information
  - typically available:
    intervals, marginal CDFs, information updates
  - typically **not** available:
    correlation information, sufficient amount of data
- Unjustified assumptions

Robust design optimization

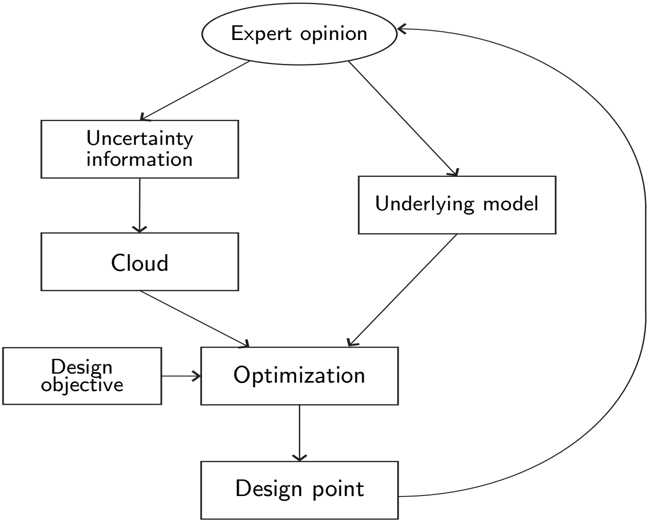
What is robust design optimization?

1. Design optimization problem
   - constrained by designated functionalities
     $\longrightarrow$ underlying system model
   - optimality with respect to a single or multiple objectives (e.g., cost, mass)
     $\longrightarrow$ objective function
2. Robustness
   - safeguard against uncertain perturbations
     $\longrightarrow$ safety constraints

Goals

- apply to real-life problems
- gather available uncertainty information
- create (or update) a corresponding **cloud**
- search for the optimal robust design
- iterate until satisfaction

Basic concept



Uncertainty model – basic idea

- choose an admissible failure probability p_f
- determine bounds on confidence regions
  - contain at least $1 - p_f$ of all possible scenarios $\varepsilon \in \mathbb{M} \subseteq \mathbb{R}^n$
  - could be computed from bounds on the CDF of ε, **but**: curse of dimensionality & lack of information
  - powerful tool in 1D: Kolmogorov-Smirnov (KS) statistics for empirical data

$\Rightarrow$ simulate data if no real data available

$\Rightarrow$ reduce the dimension to 1 by means of a potential function $V : \mathbb{R}^n \to \mathbb{R}$

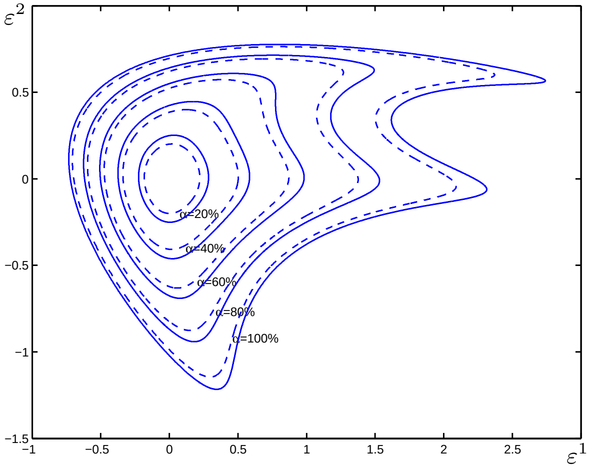
$\Rightarrow$ apply KS as in 1D case to get bounds on the CDF of $V(\varepsilon)$

$\Rightarrow$ lower and upper confidence regions for $V(\varepsilon)$

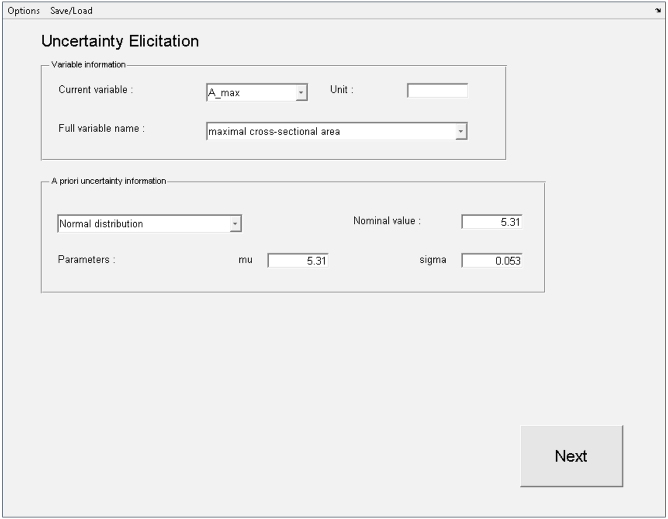
$\Rightarrow$ lower and upper confidence regions for ε as level sets of V

$\Rightarrow$ **potential cloud**

Potential cloud



A priori information



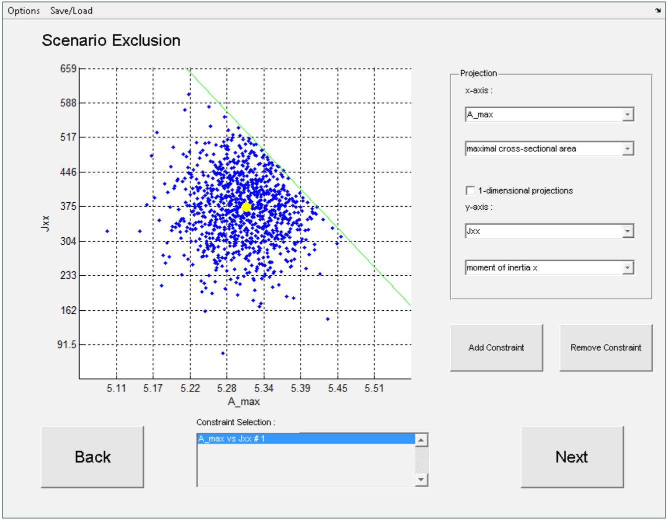
Sample generation

1. Simulate sample with respect to the a priori uncertainty information
   $\longrightarrow$ Latin hypercube sampling

2. Modify the simulated sample
   - add subjective, unformalized knowledge
   - linear scenario exclusion in 1D or 2D projections

     $\longrightarrow$ polyhedral constraints

Uncertainty elicitation ctd.



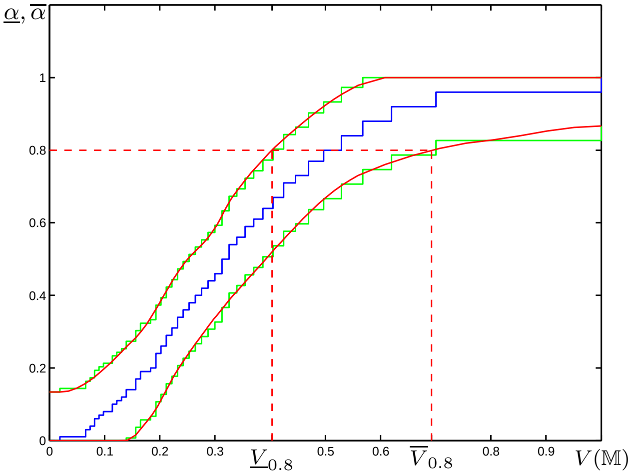
Probability bounds

- process towards probability bounds
- do **not** bound the CDF of $\varepsilon \in \mathbb{R}^n$
- **KS bounds** of the CDF of the potential $V(\varepsilon)$, i.e., $\widetilde{F} \pm d_{\mathrm{KS}}$ with

$$d_{\mathrm{KS}} = \frac{\phi^{-1}(\alpha_{\mathrm{KS}})}{\sqrt{N} + 0.12 + \frac{0.11}{\sqrt{N}}}$$

  - $\widetilde{F}$ the empirical CDF of $V(\varepsilon)$
  - ϕ the Kolmogorov function
  - α_{KS} the confidence in the KS theorem

Probability bounds ctd.



Potential clouds summary

- n-dimensional random vector ε, potential $V : \mathbb{R}^n \rightarrow \mathbb{R}$
  - our choice of V: box shaped from the a priori information
  - add the subjective cutoffs

  $\Rightarrow$ **polyhedron shape**

- lower α-cut $\underline{C}_\alpha := \{x \in \mathbb{R}^n \mid V(x) \leq \underline{V}_\alpha\}$
  contains at most a fraction of α of all possible scenarios

- upper α-cut $\overline{C}_\alpha := \{x \in \mathbb{R}^n \mid V(x) \leq \overline{V}_\alpha\}$
  contains at least a fraction of α of all possible scenarios

$\Rightarrow$ nested regions defining a **potential cloud**

Towards optimization: cloud constraints

- define the potential polyhedron shaped
- choose confidence level α
- cloud given by the regions $\underline{C}_\alpha$, $\overline{C}_\alpha$
- search for the worst-case scenario in $\underline{C}_\alpha$ or $\overline{C}_\alpha$

$\Rightarrow$ constraint in optimization problem formulation

Optimization problem formulation

$$\min_{\theta} \max_{x,z,\varepsilon} g(x) \quad \text{(objective functions)}$$

$$\text{s.t.} \quad z = Z(\theta) + \varepsilon \quad \text{(table constraints)}$$

$$G(x, z) = 0 \quad \text{(functional constraints)}$$

$$\theta \in T \quad \text{(selection constraints)}$$

$$V(\varepsilon) \leq \underline{V}_{\alpha} \quad \text{(cloud constraint)}$$

Optimization problem formulation ctd.

$$\min_{\theta} \max_{x,z,\varepsilon} g(x) \qquad \text{(objective functions)}$$

- θ design point
- x vector containing all output variables
- z vector containing all input variables
- ε random vector
- $g(x)$ design objective

Optimization problem formulation ctd.

$$\text{s.t.} \qquad z = Z(\theta) + \varepsilon \qquad \text{(table constraints)}$$

- assign to each θ a vector z of input variables
- value of z is nominal entry from $Z(\theta)$ plus its error ε

θ	**Thruster**	F/N	I_{sp}/s	m_{thrust}/kg
1	Aerojet MR-111C	0.27	210.0	200
2	EADS CHT 0.5	0.50	227.3	200
3	MBB Erno CHT 0.5	0.75	227.0	190
...				

Optimization problem formulation ctd.

$$G(x, z) = 0 \qquad \text{(functional constraints)}$$

- express the functional relationships defined in the underlying model

Optimization problem formulation ctd.

$$\theta \in T \qquad \text{(selection constraints)}$$

- T the set of all possible designs

Optimization problem formulation ctd.

$$V(\varepsilon) \leq \underline{V}_\alpha \qquad \text{(cloud constraint)}$$

- involves potential function level sets
- requires ε to be in lower α cut

Difficulties

- Mixed Integer Nonlinear Programming (MINLP)
  - potentially combinatorial explosion
- bilevel problem
  - no derivatives in outer level
  - expensive objective function
- functional constraints
  - strong nonlinearities, discontinuities
  - black box

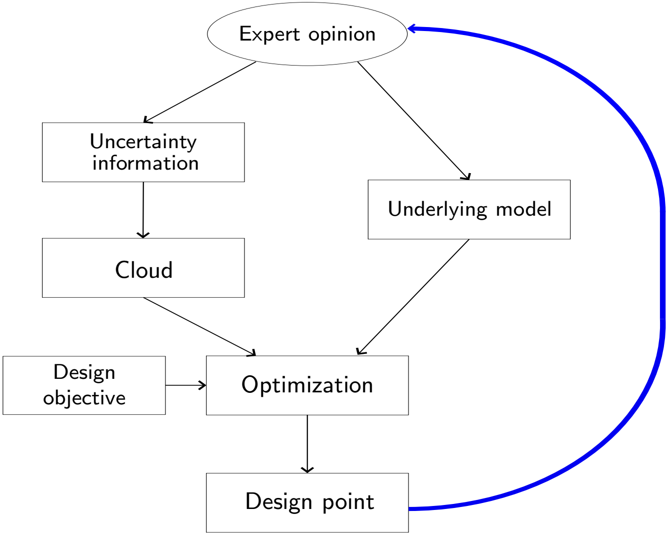
Heuristics

- inner level: solved by a linear program
- outer level: different strategies
  - SNOBFIT fits a quadratic model of the objective function and minimizes this model
  - Evolutionary algorithm
  - Separable underestimation

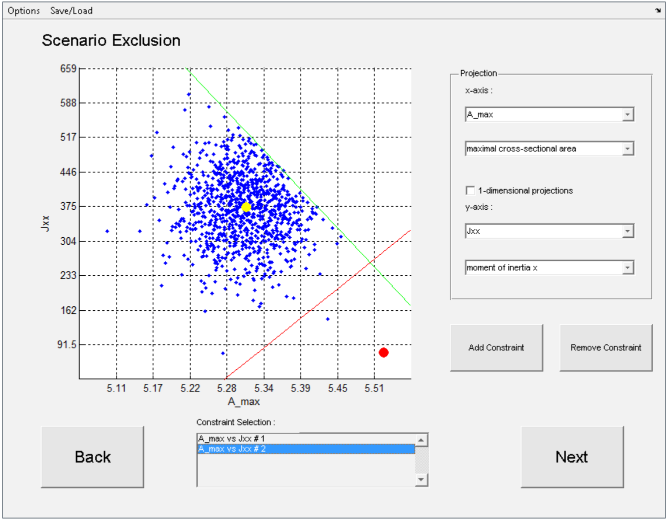
$\Rightarrow$ starting points for a local search

$\Rightarrow$ enhance chance to find the global optimum (no guarantee)

Recall: Basic concept



Uncertainty elicitation – adaptive step



Application example

Mars **E**xploration **R**over mission (**MER**)

- April 2000: official start of MER design
- June 2003: first rover launched
- July 2003: second rover launched
- November, December 2003: end of cruise stage

We focus on

- **A**ttitude **D**etermination and **C**ontrol **S**ystem (**ADCS**)

MER spacecraft

Problem specifications

- The objective is to minimize the total mass needed for the ADCS subsystem
- The decision to make is the choice of the thrusters for the ADCS from a set of available thrusters
- Total mass consists of the total fuel and the mass of the thrusters
- The fuel is computed by a MATLAB model
  $\Rightarrow$ functional constraints for the optimization

Variable structure

- fixed parameters (speed of light in vacuum, gravity constant on earth,...)
- uncertain external input variables (engine misalignment angles, spin rates,...)
- design variables (thrust F, specific impulse I_{sp}, mass m_{thrust} of a thruster)
- result variables (total mass,...)

Design table

θ	**Thruster**	F/N	I_{sp}/s	m_{thrust}/kg
1	Aerojet MR-111C	0.27	210.0	200
2	EADS CHT 0.5	0.50	227.3	200
3	MBB Erno CHT 0.5	0.75	227.0	190
4	TRW MRE 0.1	0.80	216.0	500
5	Kaiser-Marquardt KMHS Model 10	1.0	226.0	330
6	EADS CHT 1	1.1	223.0	290
7	MBB Erno CHT 2.0	2.0	227.0	200
8	EADS CHT 2	2.0	227.0	200
9	EADS S4	4.0	284.9	290
10	Kaiser-Marquardt KMHS Model 17	4.5	230.0	380
...				

Uncertainty specification

Variable	Probability Distribution	Unit
R	$N(1.3, 0.0013)$	m
δ_1	$N(0, 0.5)$	$\circ$
δ_2	$N(0, 0.5)$	$\circ$
ω_{spin0}	$N(12, 1.33)$	rpm
ω_{spin1}	$N(2, 0.0667)$	rpm
ω_{spin2}	$\Gamma(11, 0.25)$	rpm
ω_{spin3}	$L(2, 0.0667)$	rpm
ψ_{slew1}	$N(5, 0.5)$	$\circ$
ψ_{slew2}	$N(50.45, 5)$	$\circ$
ψ_{slew3}	$N(5.13, 0.5)$	$\circ$
...		

Optimization results

- Minimize the objective function: total mass m_{tot}
- Optimal design choice: $\theta = 9$

Variable	Nominal values	Worst case
m_{tot}	3.24	8.08

Optimization results

- Variation of the uncertainty information: doubled standard deviation of the normally distributed variables
- Optimal design choice: $\theta = 17$

	Nominal values	**Worst case**
m_{tot}	3.38	9.49

- originally: $\theta = 9$

$\Rightarrow$ design point sensitive to variation of uncertainty information

Optimization results

- Optimization on the nominal values:
- Optimal design choice: $\theta = 3$

	Nominal values	**Worst case**
m_{tot}	2.68	8.75

- originally: $\theta = 9$

	Nominal values	**Worst case**
m_{tot}	3.24	8.08

$\Rightarrow$ design point sensitive to accounting for uncertainty

Optimization results

- Worst-case search in 3 σ boxes:
- Optimal design choice: $\theta = 9$

	Nominal values	Worst case
m_{tot}	3.24	5.56

- originally: $\theta = 9$

	Nominal values	Worst case
m_{tot}	3.24	8.08

$\Rightarrow$ more rigorous accounting for uncertainty produces significantly different worst-case estimations

Conclusions

Clouds

- capture and process incomplete and unformalized uncertainty information
- allow for a simple uncertainty elicitation and information updating
- were applied to higher dimensional real-life design problems

These slides are available on-line at: `http://www.martin-fuchs.net`