

The Optimization Test Environment

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Motivation

Testing is crucial for users who seek to

- ① adjust solver parameters
→ improve the configuration of a particular solver
- ② compare solvers on single problems
→ find a good solution of a specific problem
- ③ compare solvers on suitable test sets
→ solver benchmarks with large test sets

The Test Environment automatizes many of these (possibly little exciting) procedures.



Study background

Comparisons of global optimization solvers:

- typically for black box problems only
- many interfaces of test environments are not very user friendly
- extensive study in 2005, based on the COCONUT Environment benchmark

! Generally, much effort has to be spent to conduct a solver comparison



Basic testing procedure

- ① organize (possibly large) test libraries
- ② solve selected problems with selected solvers
- ③ analyze, check and compare the results
- ④ create summary tables (e.g., in \LaTeX) and performance profiles



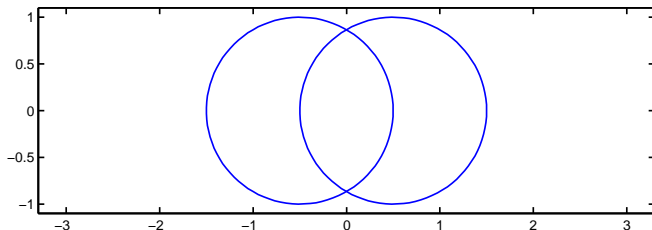
Notation for optimization problems

$$\left. \begin{array}{l} \min f(x) \\ \text{s.t. } x \in \mathbf{x}, \\ F(x) \in \mathbf{F}, \\ x_i \in \mathbb{Z} \text{ for } i \in I. \end{array} \right\} \quad (1)$$

- $\mathbf{x} = [\underline{x}, \bar{x}] = \{x \in \mathbb{R}^n \mid \underline{x} \leq x \leq \bar{x}\}$ is a **box** in \mathbb{R}^n
- $f : \mathbf{x} \rightarrow \mathbb{R}$ is the **objective function**
- $F : \mathbf{x} \rightarrow \mathbb{R}^m$ is a vector of **constraint functions**
- \mathbf{F} is a box in \mathbb{R}^m
- $I \subseteq \{1, \dots, n\}$ defines the **integer components** of x

Test problem examples

$$\left. \begin{array}{l} \min_x x_2 \\ \text{s.t. } (x_1 - 0.5)^2 + x_2^2 = 1, \\ (x_1 + 0.5)^2 + x_2^2 = 1, \\ x_1 \in [-3, 3], x_2 \in [-3, 3]. \end{array} \right\} (2)$$



$$\rightarrow \hat{x} = (0, -\sqrt{3}/2)^T \approx (0, -0.8660)^T$$



AMPL formulation

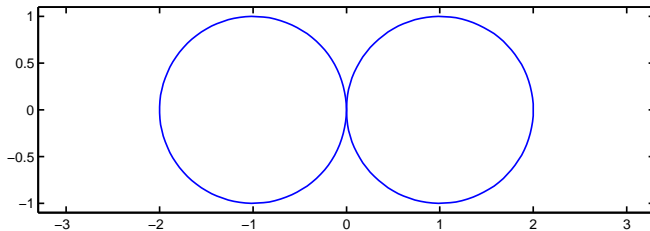
- `var x1 >=-3, <=3;`
`var x2 >=-3, <=3;`

`minimize obj: x2;`

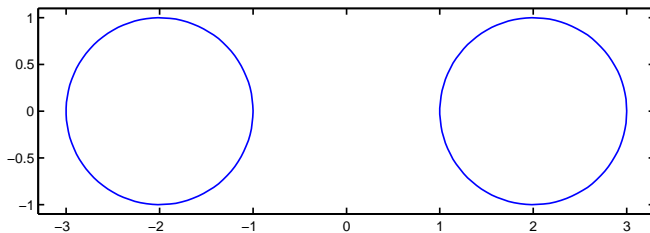
`s.t. c1: (x1-0.5)^2+x2^2=1;`
`s.t. c2: (x1+0.5)^2+x2^2=1;`
- our problem representations are DAGs (COCONUT Environment)
- converters to DAGs are available online for popular modeling languages (e.g., AMPL, GAMS)



Test problem examples ctd.



→ $x = (0, 0)^T$ is the only feasible point



→ no feasible solution



Interactive library management

[demo]



Solver interface

- ① solver selection
- ② solver configuration
→ setup paths and scripts
- ③ test problem subset selection
→ criteria editor
- ④ solve command
→ solve selected problems with selected solvers

→ [demo]



Analyze

- translate solver results to a unified format

→ .res files

modelstatus	solver model status
x(i)	solver output for $\hat{x}_i, i = 1, \dots, n$
obj	solver output for $f(\hat{x})$
infeas	feasibility distance provided by the solver
nonopt	0 if \hat{x} claimed to be at least locally optimal, 1 otherwise
time	used CPU time to solve the problem
splits	number of splits made, e.g., in branching algorithms

Solution check

- let $C := \{x \in \mathbf{x} \mid x_I \in \mathbb{Z}, F(x) \in \mathbf{F}\}$
- compute a feasibility distance d_{feas}
- intuitively: $\min_{y \in C} \|x - y\|_p$
- computationally: componentwise feasibility
- account for scaling issues via the following interval computations

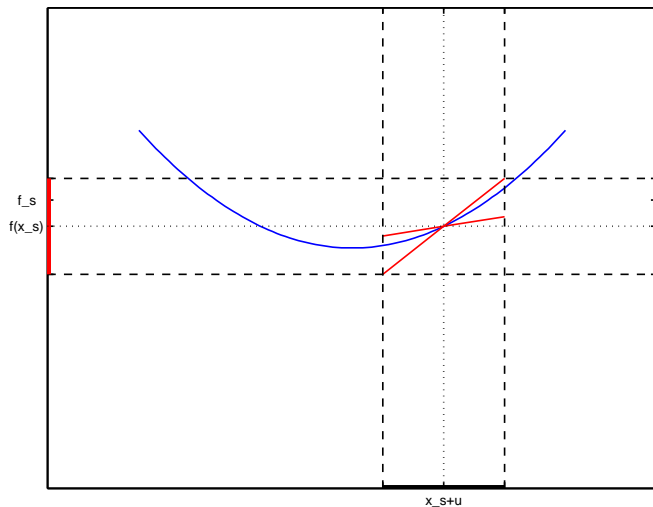


Feasibility distance

- parameters ε ($:= 10^{-6}$), κ ($:= 1$), α ($:= 0$)
- solver result x_s, f_s
- $\mathbf{u} = [-\varepsilon \max(\|x_s\|_\infty, \kappa), \varepsilon \max(\|x_s\|_\infty, \kappa)]$
- $\mathbf{x}_s = x_s + \mathbf{u}$
- objective violation
$$v_o(x_s, f_s) := \langle f(x_s) + f'(\mathbf{x}_s)(\mathbf{x}_s - x_s) - f_s \rangle$$
- constraint violations
$$v_c(x_s, f_s) := \langle F(x_s) + F'(\mathbf{x}_s)(\mathbf{x}_s - x_s) - \mathbf{F} \rangle$$
- box constraint violations
$$v_b(x_s, f_s) := \langle \mathbf{x}_s - \mathbf{x} \rangle$$
- **feasibility distance**
$$d_{\text{feas},p}(x_s, f_s) := \|(v_o(x_s, f_s), v_c(x_s, f_s), v_b(x_s, f_s))^T\|_p$$

(we use $p = \infty$)
- if $d_{\text{feas},p}(x_s, f_s) \leq \alpha \Rightarrow x_s$ **feasible**

Illustration of objective violation



Comparison of results

- $J := J_x \times J_f$, with
 $J_x := \{x_1, \dots, x_N\}$, $J_f := \{f_1, \dots, f_N\}$:
solver results that passed the check
- **global numerical solution** $(x_{\text{opt}}, f_{\text{opt}})$:
 $(x_{\text{opt}}, f_{\text{opt}}) \in J$ and $f_{\text{opt}} \leq f_j$ for all $f_j \in J_f$
- $(\tilde{x}, \tilde{f}) \in J$, $\tilde{x} \neq x_{\text{opt}}$ is also considered global if \tilde{f} is sufficiently close to f_{opt} , i.e.,

$$\tilde{f} \leq \begin{cases} f_{\text{opt}} + \beta & \text{if } |f_{\text{opt}}| \leq \kappa, \\ f_{\text{opt}} + \beta |f_{\text{opt}}| & \text{otherwise,} \end{cases}$$

with tolerance β ($:= 10^{-6}$).

- otherwise \tilde{x} is a **local numerical solution**



Comparison of results ctd.

- **best point:**

$$(x_{\text{best}}, f_{\text{best}}) = \begin{cases} (x_{\text{opt}}, f_{\text{opt}}) & \text{if } J \neq \emptyset, \\ \arg \min_{(x,f)} d_{\text{feas,p}}(x, f) & \text{if } J = \emptyset \end{cases}$$

- if the best point was found by a default local solver
→ **easy location**
- otherwise → **hard location**



Performance assessment

Count for each solver

- the number of global numerical solutions found
- the number of wrong and correct solver claims

Further performance measures for rigorous or black box solvers will be implemented in the future.



Summary tables and performance profiles

- automated creation of summary tables of the results in \LaTeX and performance profiles in JPG
- just copy paste them to result sections of scientific publications
- [demo]



Benchmark

COCONUT Environment benchmark:

- three libraries of global constrained optimization and constraint satisfaction problems
- over 1000 test problems in up to 20000 variables
- time limits depending on the problem size:

size	n	timeout (s)
1	1-9	180
2	10-99	900
3	100-999	1800
4	≥ 1000	.

We collect solver benchmark results online.



Benchmark results

baron summary statistics										
library	all	acc	wr	G+	G!	!l	W	G?	L?	!?
Lib1	264	228	38	146	82	1	34	2	0	2
Lib2	715	635	43	414	232	2	23	16	1	3
Lib3	307	274	11	252	252	5	8	0	0	3
total	1286	1137	92	812	566	8	65	18	1	8

coin summary statistics										
library	all	acc	wr	G+	G!	!l	W	G?	L?	!?
Lib1	264	239	6	138	2	0	6	0	0	0
Lib2	715	674	46	439	20	0	35	5	6	0
Lib3	307	298	3	215	3	0	2	0	1	0
total	1286	1211	55	792	25	0	43	5	7	0

Reliability analysis

solver	G+/aF+	G!/G+	wr/acc
baron	73%	69%	8%
cocos	60%	76%	12%
coin	66%	3%	4%
conopt	56%	4%	8%
knitro	49%	0%	21%
lindoglobal	67%	79%	15%
minos	58%	4%	3%
pathnlp	57%	3%	3%



Conclusions

Easy-to-use management of

- test libraries and problem subset selection
- solver configurations

Automatized

- solution check and comparison
- L^AT_EX code creation
- performance profiles



Final remarks

The benchmark results show

- BARON performed best as a global solver
- COIN performed best as a local solver

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